Let $\underset{\dim n}{X} \in \mathrm{Sm.Proj.Var}_{\mathbb{C}}$

Hodge-to-de Rham spectral sequence

$$E_2^{p,q} = H^p(X,\Omega^q)\otimes\mathbb{C} \Rightarrow H^l_{dR}(X,\mathbb{C}) = H^l_{Sing}(X,\mathbb{C}) = H^l_{Sing}(X,\mathbb{Z})\otimes_{\mathbb{Z}}\mathbb{C}$$

then X compact n-dim. smooth complex manifold

X CW-complex finite

$$\mathbb{P}^N = \mathbb{A}^N \cup \mathbb{P}^{N-1} = \mathbb{A}^N \cup \dots \cup \mathbb{A}^1 \cup (\mathbb{A}^0 = pt)$$

$\cup$

X Projective

$(e_n)^{r_n} \cup (e_{n-1})^{r_{n-1}} \cup \dots \cup$

$$X = (\mathbb{A}^n)^{r_n} \amalg (\mathbb{A}^{n-1})^{r_{n-1}} \amalg \dots \amalg (\mathbb{A}^1)^{r_1} \amalg (\mathbb{A}^0)^{r_0}$$

CW

X is irred. $r_0 \overset{!}{=} 1$

X can be modelled by homotopy theory

consider $H^l_{Sing}(X,\mathbb{Z})$

Let

$$\alpha \in N^c H^l(X,\mathbb{Z}) := \sum_{\substack{Z \hookrightarrow X \\ \text{zariski closed} \\ c\text{-codim.}}} \ker\left(H^l(X,\mathbb{Z}) \to H^l(X-Z,\mathbb{Z})\right)$$

$$= \sum_{\substack{Z \hookrightarrow X \\ \text{zariski closed} \\ c\text{-codim}}} \mathrm{Im}\left(H^l_Z(X,\mathbb{Z}) = \{\alpha \in H^l(X,\mathbb{Z}) \mid \mathrm{supp}(\alpha) \subset Z\} \to H^l(X,\mathbb{Z})\right)$$

$\beta \longmapsto f_* \beta = \alpha$

$$\widetilde{N}^c H^l(X,A) = \sum_{\substack{f:Y\to X \\ \text{Proper} \\ Y \in \mathrm{Sm.Var}_{\mathbb{C}}}} \mathrm{im}\left(f_* : H^{l-2r}(Y,A) \to H^l(X,A)\right)$$

$r \geq c$

$$\mathrm{Im}\left(H^l_{f(Y)}(X,A) \to H^l(X,A)\right)$$

$A = \mathbb{Q}$

Thm (Deligne): $\widetilde{N}^c = N^c$

$A = \mathbb{Z}$

Thm (Otteun): $\widetilde{N} \neq N$

$$\begin{tikzcd} \widetilde{Z} \arrow[d] \arrow[dr, dotted] & \\ Z \arrow[r, hook] & X \end{tikzcd}$$

$\beta \longmapsto f_* \beta$

$\alpha \qquad \alpha$

$$H^{l-2r}(\widetilde{Z},A) \to H^l(X,\mathbb{Z}) \to H^l(X-Z,\mathbb{Z})$$

$$K^M_*(F)/\ell := \underset{K_0^M}{\mathbb{Z}} \oplus F^\times \oplus F^\times \otimes_{\mathbb{Z}} F^\times / (a \otimes 1-a) \oplus \dots = H^*_{Gal}(F, \mathbb{Z}/\ell)$$

$$H^n(X, \mathbb{Z}/\ell\mathbb{Z}) \xrightarrow[\text{are zero}]{\text{all Bocksteins}} H^{n+1}(X, \mathbb{Z}/\ell\mathbb{Z})$$

$$0 \to \mathbb{Z} \xrightarrow{\ell} \mathbb{Z} \to \mathbb{Z}/\ell \to 0$$

$$H^n(F, \mathbb{Z}/\ell\mathbb{Z}) \longrightarrow H^{n+1}(F, \mathbb{Z}/\ell\mathbb{Z})$$

<u>If</u> $\alpha \in H^\ell(X, \mathbb{Z})$ torsion class

N^1

<u>then</u> $\alpha \in N^1 H^\ell(X, \mathbb{Z})$ $\quad H^n(X, \mathbb{Z}/\ell\mathbb{Z}) \xrightarrow[\text{Bockstein}]{\beta \cup (-)} H^{n+1}(X, \mathbb{Z}/\ell\mathbb{Z}) \longrightarrow H^{n+1}(X - Z, \mathbb{Z}/\ell\mathbb{Z})$

$$\alpha \in \tilde{N}^1 H^\ell(X, \mathbb{Z}) \quad H^\ell_Z(X, \mathbb{Z}) \longrightarrow H^\ell(X, \mathbb{Z}) \longrightarrow H^\ell(X - Z, \mathbb{Z})$$

$$G = (\mathbb{Z}/2\mathbb{Z})^{\times 3}$$

$$\underset{\text{infinite}}{\overset{\text{the classifying space}}{BG}} \ \underset{\text{Serre-Godaux}}{\overset{\text{Projective approximation}}{\sim\sim\sim}} \ \overset{\text{finite cw-complex}}{X}$$

$$H^i(BG, A) \underset{i<N}{=\!=} H^i(X, A)$$

$$\circlearrowleft^{\infty}$$

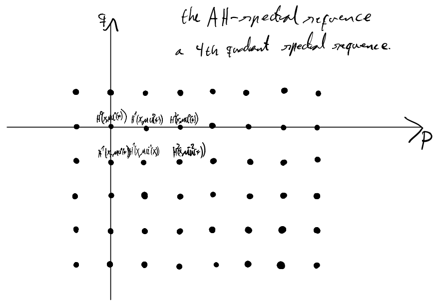
$$G'$$

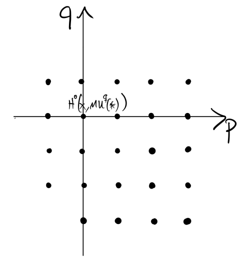
$$MU^*(X)$$

the universally oriented cohomology theory

$$E_2^{p,q} = H^p(X, MU^q(*)) \Rightarrow MU^{p+q}(X)$$

the AH-spectral sequence
a 4th quadrant spectral sequence.





Algebraic cobordism

$MU^*(-)$

a generalized cohomology theory

i.e. a spectrum.

2. Definition

In analogy with the complex cobordism spectrum MU, Voevodsky defined the algebraic cobordism spectrum MGL_S in the stable motivic homotopy category by the formula

$$\mathrm{MGL}_S = \mathrm{colim}_{n \to \infty} \Omega^n_{\mathbf{P}^1} \mathrm{Th}(V_n),$$

where S is the base scheme, $\mathrm{Th}(V_n)$ is the (infinite suspension of) the Tho space? of the tautological vector bundle V_n over the infinite Grassmannian of n-planes $\mathrm{Gr}(n)$. More precisely, $\mathrm{Gr}(n)$ is defined as the colimit of the smooth S-schemes $\mathrm{Gr}(n,k)$ as $k \to \infty$, and V_n is similarly the colimit of the tautological vector bundles over $\mathrm{Gr}(n,k)$. The notation $\Omega^n_{\mathbf{P}^1}$ denotes the nth $\mathbf{P}^1$-loop space.