Newtonian Mechanics, Special Relativity and some General Relativity.

Student Seminar

2021-04-15

Edvard Aksnes

Disclaimer: I'm not a physicist, take everything with a large grain of salt...

<u>Question</u>: How can you describe what goes on in the universe / our reality?

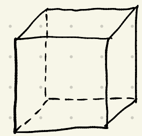
Want to predict where "things" are going.

We can **measure**:

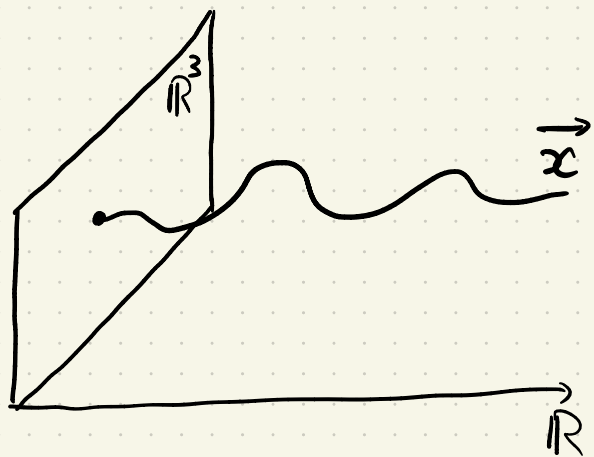
- Space 
- Time 

Seems we live in $\mathbb{R} \times \mathbb{R}^3$.

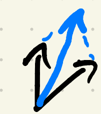
Want: How do objects move in spacetime?

Model: A big object  $\longleftrightarrow$ $\bullet$ Single point.

A motion: $\vec{x} : \mathbb{R} \longrightarrow \mathbb{R}^3$ Worldline.



Newtonian mechanics $\rightarrow$ 2 physical concepts

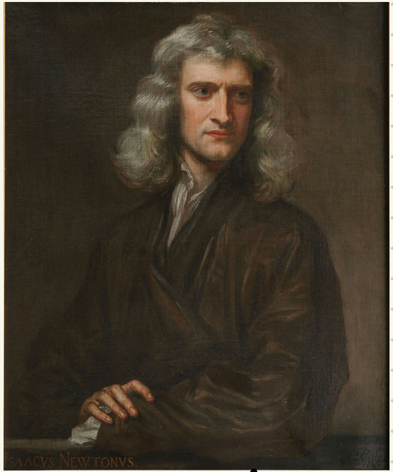
- Mass m •
- Force F ↗ ; $F + F'$ 

Defined in vague terms.

Mass = "how much <u>force</u> to move something"

Force = "something which makes something move"

Newton's Laws:



Isaac Newton

- 1st. law, principle of inertia: An object in motion stays in motion unless acted upon by a net external force

$$\sum F = 0 \iff \frac{d\vec{v}}{dt} = 0$$

- 2nd law: Time change of momentum is proportional to the force

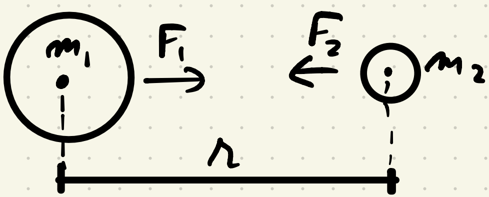
$$F = \frac{d\vec{p}}{dt} \qquad \vec{p} = m\vec{v} \quad \text{momentum}$$

- 3d law: All forces between two objects exist in equal magnitude and opposite direction: $F_A = -F_B$

How are forces determined? <u>Experiment</u>

Take an object, define its mass $= 1$. Measure how it moves under different contexts.

Example: (Newton's law of universal gravitation)



$$|F_1| = |F_2| = G \frac{m_1 m_2}{r^2}$$

gravitational constant
$\approx 6.674 \cdot 10^{-11} \frac{m^3}{kg\, s}$

Modern theory: 4 Fundamental forces

- Gravitation
- Electromagnetism
- Strong force
- Weak force

More examples: FYS-MEK1100

How to change reference frame (coord. system)?

- Spacial rotation : $(\vec{x}, t) \mapsto (R\vec{x}, t)$ $\quad R \in O(3)$
- Spacial translation : $(\vec{x}, t) \mapsto (\vec{x} + a, t)$ $\quad a \in \mathbb{R}^3$
- Uniform motion : $(\vec{x}, t) \mapsto (\vec{x} + t\vec{v}, t)$ $\quad v \in \mathbb{R}^3$

Galilean transformation: Composition of these 3.

1860's: Maxwell's equation for electromagnetism

Name	Integral equations	Differential equations
Gauss's law	$\oiint_{\partial\Omega} \mathbf{E}\cdot \mathrm{d}\mathbf{S} = 4\pi \iiint_{\Omega} \rho\, \mathrm{d}V$	$\nabla\cdot\mathbf{E} = 4\pi\rho$
Gauss's law for magnetism	$\oiint_{\partial\Omega} \mathbf{B}\cdot \mathrm{d}\mathbf{S} = 0$	$\nabla\cdot\mathbf{B} = 0$
Maxwell–Faraday equation (Faraday's law of induction)	$\oint_{\partial\Sigma} \mathbf{E}\cdot \mathrm{d}\boldsymbol{\ell} = -\frac{1}{c}\frac{\mathrm{d}}{\mathrm{d}t}\iint_{\Sigma} \mathbf{B}\cdot\mathrm{d}\mathbf{S}$	$\nabla\times\mathbf{E} = -\frac{1}{c}\frac{\partial\mathbf{B}}{\partial t}$
Ampère's circuital law (with Maxwell's addition)	$\oint_{\partial\Sigma} \mathbf{B}\cdot \mathrm{d}\boldsymbol{\ell} = \frac{1}{c}\left(4\pi\iint_{\Sigma}\mathbf{J}\cdot\mathrm{d}\mathbf{S} + \frac{\mathrm{d}}{\mathrm{d}t}\iint_{\Sigma}\mathbf{E}\cdot\mathrm{d}\mathbf{S}\right)$	$\nabla\times\mathbf{B} = \frac{1}{c}\left(4\pi\mathbf{J} + \frac{\partial\mathbf{E}}{\partial t}\right)$

James Clerk Maxwell

$$\mathbf{F} = q(\mathbf{E} + \mathbf{v}\times\mathbf{B})$$ Lorentz force

$\Longrightarrow$ NOT invariant under Galilean transformation

<u>However</u>: Invariant under Lorentz transformation:

Velocity: $\vec{v} = (u, 0, 0)$

$$\begin{pmatrix} t \\ x \\ y \\ z \end{pmatrix} \mapsto \begin{pmatrix} \gamma\left(t - \frac{ux}{c^2}\right) \\ \gamma(x - ut) \\ y \\ z \end{pmatrix}$$

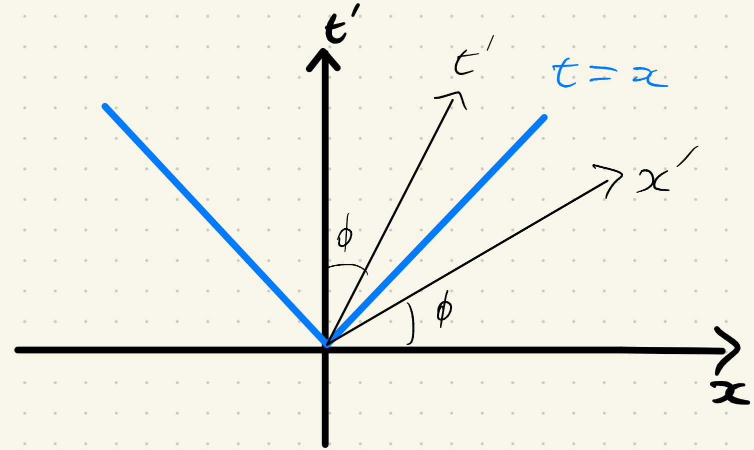
$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Lorentz factor.

Observations:

1. Time dilation.

2. No simultaneity.
   $\{t = 0\} \neq \{t' = 0\} \subseteq$ "$\mathbb{R}^4$"



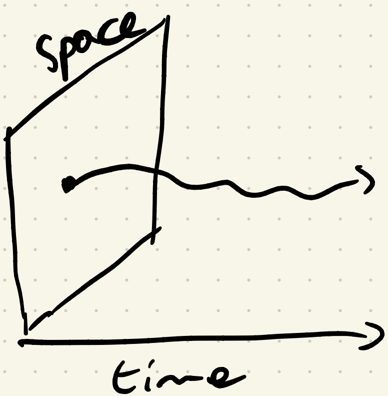
Einstein :

Galilean transformations

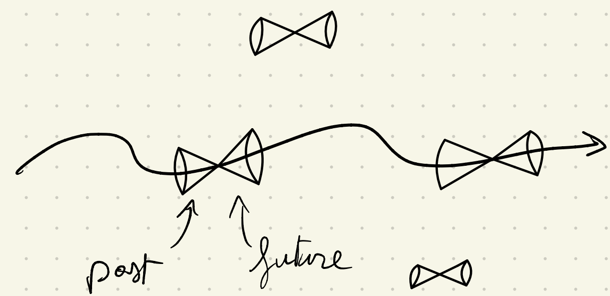
$$\Downarrow$$

Poincaré transformations (Lorentz tr. + translation / rotation)

Albert Einstein



$\Longrightarrow$

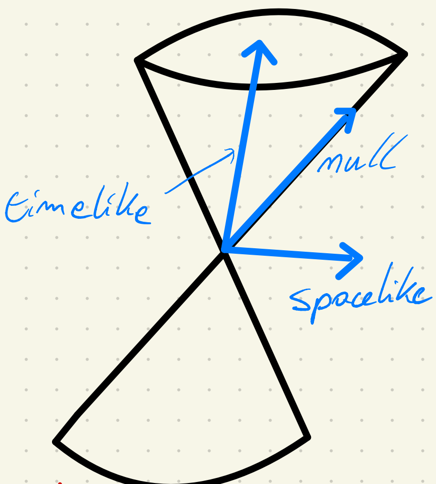


SpaceTime $\mathbb{R}^{1,3}$

Spacetime / Minkowski Space :

4 manifold $\mathbb{R}^{1,3} = (\mathbb{R}^4, d)$

where $d\big((t_1,\vec{x}_1),(t_2,\vec{x}_2)\big) = \sqrt{-c^2(t_1-t_2)^2 + \|\vec{x}_1-\vec{x}_2\|^2}$



lightcone

Metric: $\eta_{\mu\nu} = \begin{pmatrix} -c^2 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

local coords: $(x^\mu)_{\mu=0,\dots,3}$

A particle with mass follows a worldline where all tangent vectors are timelike. $\tilde{x}:[0,1]\to\mathbb{R}^{1,3}$, param λ

$$\Delta\tau = \int_0^1 d\tau = \int_0^1 \sqrt{\sum_{\mu,\nu=0}^{3} -\eta_{\mu\nu}\frac{dx^\mu}{d\lambda}\frac{dx^\nu}{d\lambda}}\,d\lambda$$

Given a particle with mass, we can parametrize its path with the internal clock $\Delta\tau$.

$\rightsquigarrow$ Four-velocity $\qquad U = \left(\frac{d\tilde{x}^\mu}{d\tau}\right)_{\mu=0,\dots,3} \qquad \|U\| = c^2$

$\rightsquigarrow$ Momentum four-vector $\qquad p = mU$

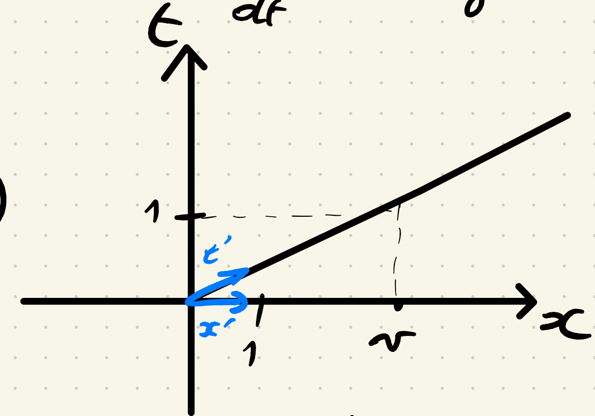
m mass of particle.

Energy is the 0-th component of p. $E = p^0$.

In the reference frame of the particle $U = (c^2, 0, 0, 0)$

$\Rightarrow \quad E = mc^2$

Suppose we have a reference frame $(x^\mu)_{\mu=0,\dots,3}$, and a particle moving with velocity $v = \frac{dx}{dt}$ along the x axis.



In particle ref frame: $(t', x') = (\tau, 0)$

$$p = (mc^2, 0, 0, 0)$$

In our ref frame: (t, x)

Apply Lorentz transform
$$x = \gamma(x' + vt') = \gamma v t'$$
$$t = \gamma\left(t' + v\frac{x'}{c^2}\right) = \gamma t'$$

$$p = (\gamma m c^2, v\gamma m, 0, 0) \qquad \gamma = \sqrt{1 - \frac{v^2}{c^2}}$$

Small $v \rightsquigarrow p^0 \approx m + \frac{1}{2}mv^2$ (rest energy + potential), $p^1 = mv$ (Newtonian momentum)

Gravity is special

Inertial mass

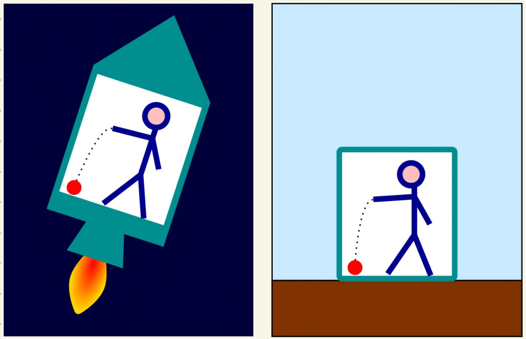
$$[F = m_i a]$$

Gravitational Mass.

$$[F_g = -m_g \nabla \phi]$$

Weak Equivalence Principle:

$$m_i = m_g$$

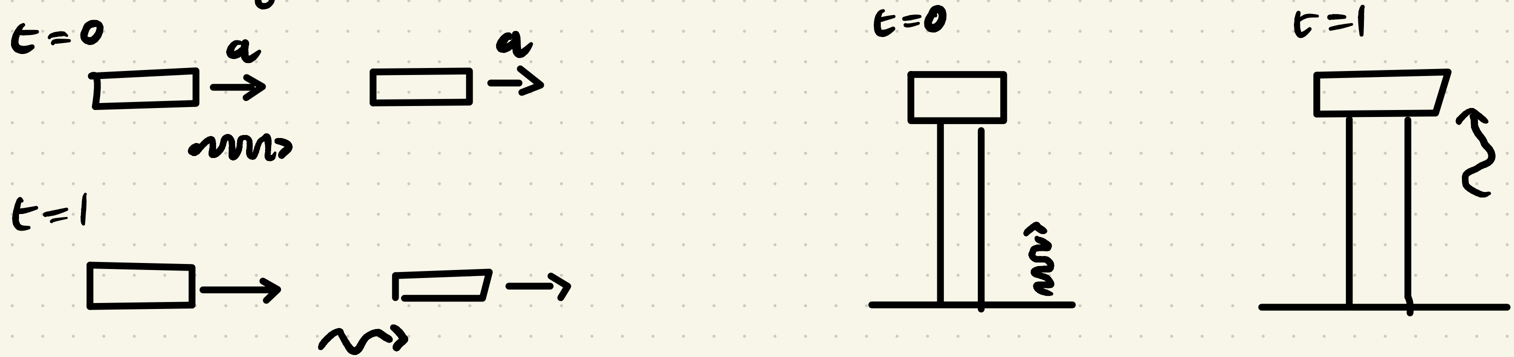


Experimentally verified

⇝ Einstein Equivalence Principle

In small enough regions of spacetime, the laws of physics reduce to those of special relativity. It is impossible to detect the existence of a gravitational field.

Moving Rockets vs Tower



$\rightsquigarrow$ Spacetime is not $\mathbb{R}^{1,3}$,
it must be curved!

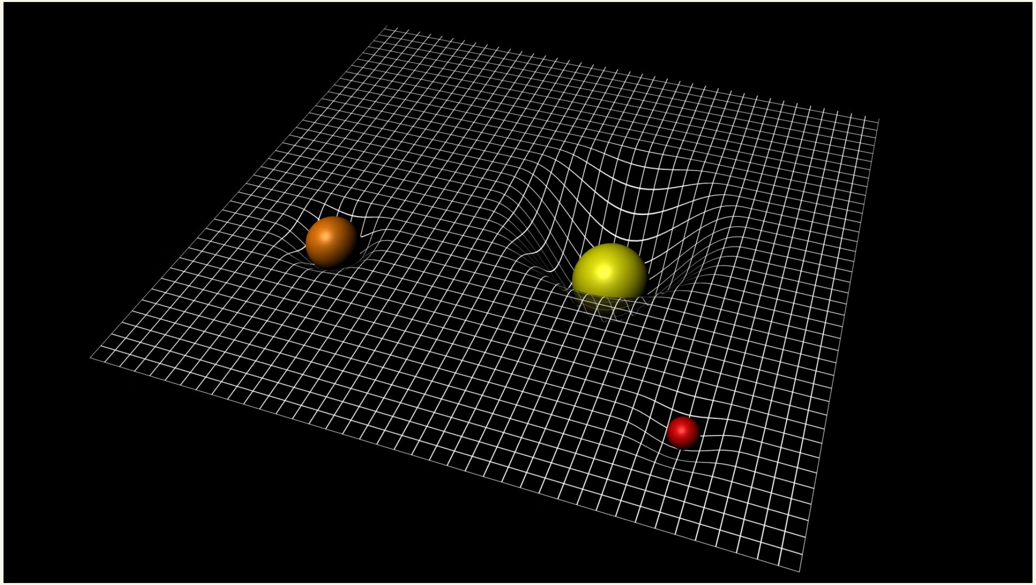
In 1915, having worked for 8 years, Einstein published the General theory of relativity.

Curvature of spacetime $\longleftrightarrow$ Matter & Energy

Einstein's field equation: $R_{\mu\nu} - \frac{1}{2} R\, g_{\mu\nu} = 8\pi G T_{\mu\nu}$

Free particles: Geodisic equation: $\sum_{\rho,\sigma} \frac{d^2 x^\mu}{d\lambda^2} + \Gamma^\mu_{\rho\sigma} \frac{dx^\rho}{d\lambda} \frac{dx^\sigma}{d\lambda} = 0$

Thanks for your time!



© ESA